Exploiting Derivative Rules

Every differentiation rule $F'(x) = f(x)$ corresponds to a rule for finding the anti-derivative $F(x)$ of some function f.

a) Find an anti-derivative rule that is the inverse of the sum rule $(f + g)'(x) = f'(x) + g'(x)$.

b) Find an anti-derivative rule that is the inverse of the product rule $(f \cdot g)'(x) = f(x)g'(x) + f'(x)g(x)$.

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a) $(f+g)'(x) = f'(x) + g'(x)$

$$\int (f+g)'(x)\,dx = \int \left(f'(x) + g'(x)\right) dx$$

$$= \int f'(x)\,dx + \int g'(x)\,dx$$

$$= \int d(f(x)) + \int d(g(x))$$

$$= f(x) + g(x) + C$$

b) $(f \cdot g)'(x) = f(x)g'(x) + f'(x)g(x)$

$$\Rightarrow \int (f \cdot g)'(x)\,dx = \int \left(f(x)g'(x) + f'(x)g(x)\right) dx$$

$$= \int d\left(f(x)g(x)\right)$$

$$= f(x)g(x) + C$$